KENDRIYA VIDYALAYA SANGATHAN

MODEL PAPER -3

CLASS 12 – MATHEMATICS

me Allowed: 3 hours | Maximum Marks: 80

eneral Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with subparts.

SECTION A

1. The vector equation of the x-axis is given by

   (a) $\vec{r} = \hat{j} + \hat{k}$ (b) $\vec{r} = \hat{i}$ (c) $\vec{r} = \lambda \hat{i}$ (d) none of these

2. $\int \frac{dx}{\sqrt{\sin^3 x \cos x}}$

   (a) $-\frac{2}{\sqrt{\tan x}} + c$ (b) $2\sqrt{\cot x} + c$ (c) $2\sqrt{\tan x} + c$ (d) $-2\sqrt{\tan x} + c$

3. What is the value of p for which the vector p(2i - + 2) is of 3 units length?

   (a) 3 (b) 6 (c) 1 (d) 2

4. The area of the region bounded by the curve y = sin x between the ordinates x = 0, x = and the x-axis is

   (a) 2 sq units (b) 4 sq units (c) 1 sq units (d) 3 sq units

5. The area bounded by the parabola y = x2 and the line y = x is

(a) 1/2 sq units (b) none of these (c) 1/6 sq units (d) 1/ 3 sq units

6. Let A and B be two events. If P (A) = 0.2, P (B) = 0.4, P (A⋃B) = 0.6, then P (A|B) is equal to

(a) 0.5 (b) 0.8 (c) 0.3 (d) 0

7. Two events E and F are independent. If P(E) = 0.3, P(E⋃F)=0.5 then P(E|F) - P(F|E) equals

(a) 3/35 (b) 2/7 (c) 1/7 (d) 1/70

8. The objective function Z = 4 x + 3 y can be maximised subjected to the constraints $3x + 4y \leq 24$, $8x + 6y \leq 48$, $x \leq 5$, $y \leq 6$; $x, y \geq 0$

(a) at only one point b) None of these c) at two points only

d) at an infinite number of points

9. Show that the points A(1, – 2, – 8), B (5, 0, – 2) and C (11, 3, 7) are collinear, and find the ratio in which B divides AC.

(a) 3 :2 (b) 2 :4 (c) 2 :3 (d) 2 :1

10. Consider a differential equation of order m and degree n. Which one of the following pairs is not feasible?

(a) (2 ,3/2) (b)(2 ,4) (c) (3 , 2) (d) (2 ,2)

11. Integrating factor of the differential equation cos x + y sin x = 1 is

(a) Tan x (b) sin x (c) sec x (d

12. $\int \frac{\sin\sqrt{x}}{\sqrt{x}}dx$

(a) $-2\cos\sqrt{x}+c$ (b) $2\cos\sqrt{x}+c$ (c) $-\frac{\cos\sqrt{x}}{2}+c$ (d) $\frac{\cos\sqrt{x}}{2}+c$

13. What is the general solution of the differential equation xdy - y dx = y^2 ? Where, C is an arbitrary constant.

(a) None of these (b) x = cy (c) x +xy –cy = 0 (d) y^2 = cx

14. The function f(x) = cot x is discontinuous on the set

(a) $\left\{x=(2n+1)\frac{\pi}{2};n\in Z\right\}$ (b) $\{x=(2n\pi:n\in Z\}$

c) $\{x=(n\pi/2; n\in Z\}$ (d) $\{x=(n\pi : n\in Z\}$

15. If A is a non singular matrix of order 3, then $|adj(A^3)|=$

(a) none of these (b) $|A|^8$ (c) $|A|^6$ (d) $|A|^9$

16. If m and n are the order and degree of the differential equation,

$$(y_2)^5+\frac{4(y_z)^3}{(y_3)}+(y_3)=x^2-1,$$, then

(a) m = 3, n = 5 (b) m = 3, n = 3

(c) m = 3, n = 1 (d) m = 3, n = 2

17. The principal value of $\cot^{-1}(-\sqrt{3})$ is

(a) $2\pi/6$ (b) $5\pi/6$ (c) $\pi/6$ (d) $7\pi/6$

18. Let A = (6, 3, -2), B = (2, 4, -3), C = (1, 5, 2), D = (1, 6, m), where m > 0. The shortest distance between AB and CD is 3. Then the value of m is:

(a) 4 (b) 6 (c) 2 (d) 3

19. Assertion (A): The maximum value of Z = x + 3y. Such that $2x + y \leq 20$, $x + 2y \leq 20$, x, y ≥ 0 is 30.

Reason (R): The variables that enter into the problem are called decision variables.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

20. Assertion (A): Every differentiable function is continuous but converse is not true.

Reason (R): Function f(x) = |x| is continuous.

(i) Both A and R are true and R is the correct explanation of A.
(j) Both A and R are true but R is not the correct explanation of A.
(k) A is true but R is false.
(l) A is false but R is true.

SECTION –B

21. Solve the differential equation: $y\,(1 + e^x)\,dy = (y + 1)\,e^x\,dx$.

22. At what points on the curve, is the tangent parallel to the x-axis? $y = x^2$ on $[-2, 2]$.

23. Find the shortest distance between the lines

$$\vec{r} = (6\hat{i} + 2\hat{j} + 2\wedge) + \lambda(\hat{i} - 2\hat{j} + 2\hat{k}) \;\&\; \vec{r} = -4\hat{i} - \hat{k} + \mu(3\hat{i} - 2\hat{j} - 2\hat{k})$$

OR

By computing the shortest distance determine the pairs of lines intersect or not:

$$\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k}) \;\&\; \vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} - \hat{j} + 2\hat{k})$$

24. In a class, 40% students study mathematics; 25% study biology and 15% study both mathematics and biology. One student is selected at random. Find the probability that he studies mathematics if it is known that he studies biology.

25. Find the value of $\cos^{-1}(1/2) + 2\sin^{-1}(1/2)$.

SECTION – C

26. Minimise $Z = 13x - 15y$, subject to the constraints: $x + y \leq 7$, $2x - 3y + 6 \geq 0$, $x \geq 0$, $y \geq 0$.

27. Find the area bounded by the line $y = x$, the x-axis and the ordinates $x = -1$, $x = 2$.

OR

Find the area enclosed by the circle $x^2 + y^2 = a^2$

28. Evaluate $\int \frac{dx}{x(x^5 + 3)}$ OR Evaluate $\int \sin(\log x)\,dx$.

29. Find the direction cosines of the sides of the triangle whose vertices are (3, 5, -4), (-1, 1, 2) and (-5, -5, -2). **OR**

Find the perpendicular distance of the point (2, 3, 4) from the line $\frac{4-x}{2}=\frac{y}{6}=\frac{1-z}{3}$ Also find coordinates of foot of perpendicular.

30. Find the area of the region lying in the first quadrant and bounded by $y = 4x^2$, $x = 0$, $y = 1$ and $y = 4$.

31. Find $\frac{dy}{dx}$ if $(\cos x)^y = (\cos y)^x$

SECTION – D

32. If with reference to the right handed system of mutually perpendicular unit vectors $\hat{i}, \hat{j}$ and $\hat{k}$, $\vec{\alpha} = 3\hat{i} - \hat{j}$ and $\vec{\beta} = 2\hat{i} + \hat{j} - 3\hat{k}$. Express $\vec{\beta}$ in the form of $\vec{\beta} = \vec{\beta}_1 + \vec{\beta}_2$ where $\vec{\beta}$ is parallel to $\vec{\beta}_1$ is parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ is perpendicular to $\vec{\alpha}$.

33. The cost of 4kg onion, 3kg wheat and 2kg rice is Rs. 60. The cost of 2kg onion, 4kg wheat and 6kg rice is Rs. 90. The cost of 6kg onion 2kg wheat and 3kg rice is Rs. 70. Find the cost of each item per kg by matrix method.

OR

The sum of three numbers is 6. If we multiply third number by 3 and add second number to it, we get 11. By adding first and third number, we get double of the second number. Represent it algebraically and find the numbers using matrix method.

34. Let A = {1, 2, 3, ...9} and R be the relation in A A defined by (a, b) R (c, d) if a + d = b + c for (a, b), (c, d) in A x A. Prove that R is an equivalence relation and also obtain the equivalence class [(2, 5)]. **OR**

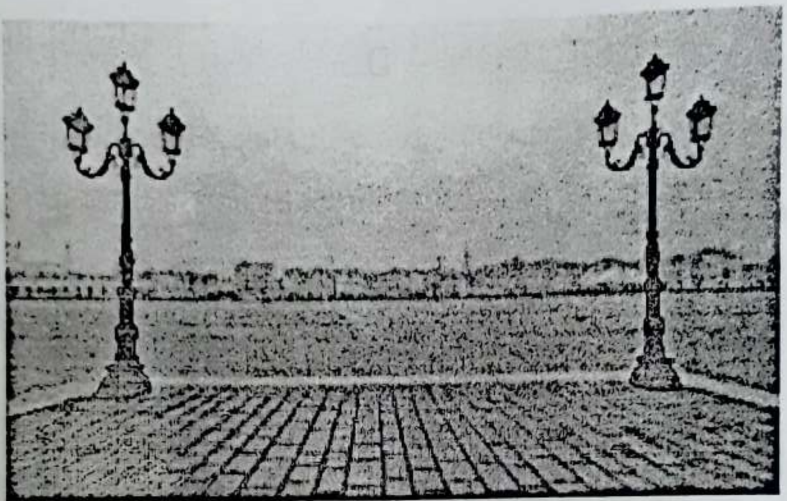
Let A = R – {3} and B = R – {1}. Consider the function f: A → B defined by .f(x) = $\left(\frac{x-2}{x-3}\right)$. Show that f is one- one and onto .

35. Integrate the function 1 /$(x-x^3)$

SECTION – E

36. **Read the text carefully and answer the questions:**

In a street two lamp posts are 600 feet apart. The light intensity at a distance d from the first (stronger) lamp post is $1000/d^2$, the light intensity at distance d from the second (weaker) lamp post is $125/d^2$ (in both cases the light intensity is inversely proportional to the square of the distance to the light source). The combined light intensity is the sum of the two light intensities coming from both lamp posts.



(i) If I(x) denotes the combined light intensity, then find the value of x so that I(x) is minimum.

(ii) Find the darkest spot between the two lights.

(iii) If you are in between the lamp posts, at distance x feet from the stronger light, then write the combined light intensity coming from both lamp posts as function of x.

OR

Find the minimum combined light intensity?

37. **Read the text carefully and answer the questions:**

Consider 2 families A and B. Suppose there are 4 men, 4 women and 4 children in family A and 2 men, 2 women and 2 children in family B. The recommended daily amount of calories is 2400 for a man, 1900 for a woman, 1800 for children and 45 grams of proteins for a man, 55 grams for a woman and 33 grams for children

(i) Represent the requirement of calories and proteins for each person in matrix form.

(ii) Find the requirement of calories of family A and requirement of proteins of family B.

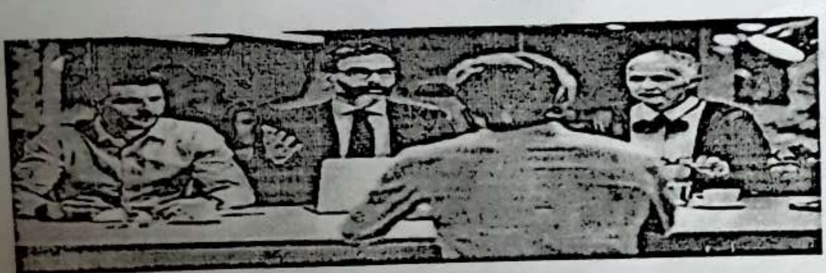
(iii) Represent the requirement of calories and proteins If each person increases the protein intake by 5% and decrease the calories by 5% in matrix form.

OR

If A and B are two matrices such that AB = B and BA = A, then find $A^2 + B^2$ in terms of A and B.

38. Read the text carefully and answer the questions:

To hire a marketing manager, it's important to find a way to properly assess candidates who can bring radical changes and has leadership experience. Ajay, Ramesh and Ravi attend the interview for the post of a marketing manager. Ajay, Ramesh and Ravi chances of being selected as the manager of a firm are in the ratio 4 : 1 : 2 respectively. The respective probabilities for them to introduce a radical change in marketing strategy are 0.3, 0.8, and 0.5. If the change does take place.



(iii) Find the probability that it is due to the appointment of Ajay (A).

(iv) Find the probability that it is due to the appointment of Ramesh (B)

BLUE PRINT (MODEL PAPER-3)

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

Chapter Name	**Assertion & Reason**	**Case Study Questions**	**Multiple Choice Question**	**Subjective Questions**			**Total**
				2 Marks	**3 Marks**	**5 Marks**	
Relations and Functions						1(5)*	1(5)
inverse Trigonometric Functions			1(1)	1(2)			2(3)
Matrices		1(4)					1(4)
Determinants			1(1)			1(5)*	2(6)
Continuity and Differentiability	1(1)		1(1)	1(2)	1(3)		4(7)

Application of Derivatives		1(4)					1(4)
Integrals				2(1)	1(3)*	1(5)	4(10)
Application of Integrals				2(1)	2(3)*		4(8)
Differential Equations				4(1)	1(2)		5(6)
Vector Algebra				2(1)		1(5)	3(7)
Three Dimensional Geometry				2(1)	1(2)*	1(3)*	4(7)
Linear Programming	1(1)			1(1)		1(3)	3(5)
Probability		1(4)		2(1)	1(2)		4(8)
Total	2(2)	3(12)	18(18)	5(10)	6(18)	4(20)	38(80)

MARKING SCHEME OF MODEL PAPER 3

CLASS XII, MATHEMATICS

Question number		Value points/Answer	Marks
1	d	$\vec{r} = \lambda i \wedge$	1
2	a	$-\frac{2}{\sqrt{\tan x}}+C$	1
3	c	1	1
4	c	1	1

5	c	1/6	1
6	d	0	1
7	d	1/70	1
8	d	at an infinite number of point	1
9	c	2:3	1
10	a	(2, 2/3)	1
11	c	secx	1
12	a	$-2\cos\sqrt{x}+c$	1
13	b	X= cy	1
14	d	$\{x=n\pi : n\in Z\}$	1
15	c	$\|A\|^6$	1
16	d	m = 3, n = 2	1
17	b	$5\pi/6$	1
18	d	3	1
19	b	Both A and R are true but R is not the correct explanation of A.	1
20	c	A is true but R is false	1
21		y - log |y + 1| = log |1 + e^x | + C	2
22		The point at which the curve has tangent parallel to x – axis is (0,0).	2
23		$3/\sqrt{2}$ or the given lines do not intersect.	2
24		3/5	2
25		$2\pi/3$	2
26		the minimum value is -30 at the point (0, 2).	3
27		the required area is 5/2 sq units or π a^2	3

$\frac{1}{15}\log\left(x^5/\ x^5+3\right)+c$ or $x/2\ [\sin\log x - \cos\log x] + c$ 3

14 units. 3

7/3 sq. units 3

$$\log\cos x\cdot\frac{dy}{dx}+y\cdot\frac{d}{dx}(\log\cos x)=\log\cos y\cdot\frac{d}{dx}(x)+x\cdot\frac{d}{dx}(\log\cos y)$$

$$\Rightarrow \log\cos x\frac{dy}{dx}+y\cdot\frac{1}{\cos x}\cdot\frac{d}{dx}(\cos x)=\log\cos y\cdot 1+x\cdot\frac{1}{\cos y}\cdot\frac{d}{dx}(\cos y)$$

$$\Rightarrow \log\cos x\frac{dy}{dx}+\frac{y}{\cos x}\cdot(-\sin x)=\log\cos y+\frac{x}{\cos y}(-\sin y)\cdot\frac{dy}{dx}$$

$$\Rightarrow \log\cos x\frac{dy}{dx}-y\tan x=\log\cos y-x\tan y\frac{dy}{dx}$$ 3

31

$$\Rightarrow (\log\cos x+x\tan y)\frac{dy}{dx}=y\tan x+\log\cos y$$

$$\therefore \frac{dy}{dx}=\frac{y\tan x+\log\cos y}{x\tan y+\log\cos x}$$

5

32 Let $\vec{\beta}_1=\lambda\vec{\alpha}, \lambda\ is\ a\ scalar, i.e.\ \vec{\beta}_1=3\lambda\hat{i}-\lambda\hat{j}$

Now $\vec{\beta}_2=\vec{\beta}-\vec{\beta}_1=\ (2-3\lambda)\hat{i}+(1+\lambda)\hat{j}-3\hat{k}$

Now, since $\vec{\beta}_2$ is to be perpendicular to $\vec{\alpha}$, we should have $\vec{\alpha}.\vec{\beta}_2=0\ i.e.,$

$$3(2-3\lambda)-(1+\lambda)=0$$

or

$$\lambda=\frac{1}{2}$$

Therefore $\vec{\beta}_1=\frac{3}{2}\hat{i}-\frac{1}{2}\hat{j}\ and\ \vec{\beta}_2=\frac{1}{2}\hat{i}+\frac{3}{2}\hat{j}-3\hat{k}$

5

33 x = 5, y = 8, z = 8 or , x=1, y=2, z=3

34 equivalence class of (2,5) is {(1,4),(2,5),(3,6),(4,7),(5,8),(6,9).

or correct proof 5

35 $1/2\log\left|\frac{x^2}{1-x^2}\right|+c$ 5

36 (i) I(x) is minimum when you are at 400 feet from the strong intensity lamp post. 1

(ii) 200 feet from the weaker lamp post. 1

$\frac{1000}{x^2}+\frac{125}{(600-x)^2} or 3/320$ units 2

37

(i) F = $\begin{bmatrix} 4 & 4 & 4 \\ 2 & 2 & 2 \end{bmatrix} and\, R = \begin{bmatrix} 2400 & 45 \\ 1900 & 55 \\ 1800 & 33 \end{bmatrix}$

(ii) FR = $\begin{bmatrix} 24400 & 532 \\ 12200 & 266 \end{bmatrix}$

(iii) $\begin{bmatrix} 2280 & 45.25 \\ 1805 & 55.75 \\ 1710 & 34.65 \end{bmatrix}$ or A + B

38 (i) 2/5 2

(ii) 4/15 2

MODEL SAMPLE PAPER

CLASS XII SESSION: 2022-23

MATHEMATICS (CODE-041)

Time Allowed: 3 Hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains- Five Sections A , B , C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion – Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)- type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)- type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)- type questions of 5 marks each.
6. Section E has 3 source based/case based / passage based / integrated unit of assessment (4marks each) with sub parts.

SECTION A

(MULTIPLE CHOICE QUESTIONS)

EACH QUESTIONS CARRIES 1 MARK

Q1. A relation R in a set A={1,2,3} defined as R = { (1,2) }, then

(a) R is Reflexive (b) R is Symmetric (c) R is Transitive

(d) R is an Equivalence Relation.

Q2. A function f : N $\rightarrow$N defined by f(x)= $x^2 + x + 1$ is

(a) One – one (b) Many- one (c) Onto (d) Bijective

Q3. Principal value of $\tan^{-1}\sqrt{3}-\cot^{-1}\left(-\sqrt{3}\right)$ is

(a) 0 (b) π (c) $\frac{\pi}{2}$ (d) $-\frac{\pi}{2}$

Q4. If A = $[a_{ij}]$ is a 2 × 3 matrix, such that $a_{ij} = \frac{(-1+2j)^2}{5}$. then a_{23} is

(a)) $\frac{1}{5}$ (b)) $\frac{2}{5}$ (c) $\frac{9}{5}$ (d)) $\frac{16}{5}$

Q5. If A is a square matrix of order 3 × 3 such that $|A| = 2$ then $|A(adjA)|$

(a) 4 (b) 16 (c) 8 (d) 0

Q6. If for any square matrix A , A(adjA) = $\begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$ then value of $|A|$

(a) 3 (b) 2 (c) 8 (d) 1

Q7. The value of 'a' for which the function $f(x) = \begin{cases} x+1, x \le 1 \\ 3-ax^2, x > 1 \end{cases}$ is continuous.

(a) 0 (b) -5 (c) 4 (d) 1

Q8. If a function f(x) is defined as $f(x) = \begin{cases} ax+1, if\ x \le 3 \\ bx+3, if\ x > 3 \end{cases}$ is continuous then the value of 3(a – b) is

(a) 0 (b) 2 (c) -1 (d) 1

Q9. Derivative of $\sec^{-1}\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}\right) + \sin^{-1}\left(\frac{\sqrt{x}-1}{\sqrt{x}+1}\right)$ w.r.t x is

(a) $\frac{\pi}{2}$ (b) 0 (c) 1 (d) $\sqrt{\ }2$

Q10. If $y = \tan^{-1}\left(\frac{1-x^2}{1+x^2}\right) then \frac{dy}{dx}$ is

(a) $\frac{1}{x}$

(b) $\frac{\pi}{4}-x^2$

(c) $-\frac{2x}{1+x^4}$

(d) $\frac{2x}{1+x^4}$

Q11. If $\int \frac{\sin x}{\sin(x-a)}dx = Ax + B\log|\sin(x-a)| + C$ then the value of $A^2+ B^2$ is

(a) 2x

(b) 0

(c) 1

(d) π

Q12 Value of . $\int_{-10}^{10} \log\left(\frac{2+x}{2-x}\right)dx$ is

(a) -x (b) 0 (c) 2 (d) $-\pi$

Q13.If m is the order and n is the degree of given differential equation,

: $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^3 + x^4 = 0$ then what is the value of m + n

(a) 5

(b) 9

(c) 4

(d) 7

Q14. If $x(\hat{i}+\hat{j}+\hat{k})$ is a unit vector then the value of x is

(a) $\frac{1}{3}$

(b) $\frac{-1}{2}$

(c) $\pm\frac{1}{\sqrt{3}}$

(d) $\sqrt{3}$

Q15. What is the value of λ if projection of $\vec{a} = \lambda\hat{i} + \hat{j} + 4\hat{k}$ on $\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$ is 4 units

(a) 5 (b) 5 or -23

(c) -5 or 23 (d) 4

Q16. The value of $\hat{i}(\hat{j} \times (\hat{k}) + \hat{j}.(\hat{k} \times \hat{i}) + \hat{k}.(\hat{i} \times \hat{j})$

(a) 1 (b) 3

(c) 0 (d) -1

Q17. The direction ratio of the line $\frac{3-2x}{4} = \frac{y+5}{3} = \frac{6-z}{6}$ is

(a) 4, 3, 6

(b) 2, 3, - 6

(c) 2, -3, 6

(d) – 2, -3, -6

Q18. A couple has 2 children. Then probability that both are boys, if it is known that one of the children is a boy is ?

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$

(c) $\frac{2}{3}$ (d) 1

ASSERTION- REASON BASED QUESTIONS

In the following questions a statement of Assertion (A) is followed by statement of Reason (R).

Choose the correct answer out of the following choices.

(a) Both A and R are true, and R is the correct explanation of A.

(b) Both A and R are true, but R is not the correct explanation of A.

(c) A is true, R is false.

(d) A is false, R is true.

Q19. Assertion (A): The value of integral $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}(x^3 + x\cos x + \tan^3 x + 1)dx$ *is* 1

Reason (R) : If f(x) is odd function then $\int_{-a}^{a} f(x)dx = 0$

Q20. Assertion (A): Two lines $\frac{1+x}{3} = \frac{7y+14}{2p} = \frac{z-3}{2}$ and $\frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$

Are at right angles if p = $\frac{70}{11}$

Reason (R) : Two lines having direction ratio a_1, b_1, c_1 and a_2, b_2, c_2 are perpendicular if $a_1a_2 + b_1b_2 + c_1c_2 = 0$

SECTION-B

This section comprises of very short answer type question (VSA) of 2 marks each

Q21. Find the value of $\cos^{-1}\left[\cos(\frac{13\pi}{4})\right]$

OR

Prove that a function f: R $\rightarrow$ R defined by f(x) = [x] , where [ ] denotes greatest integral function is many one function. Justify your answer.

Q22. If A and B are symmetric matrices then prove that AB – BA is skew symmetric matrix.

Q23. A stone is dropped into calm lake and waves moves in circles at a speed of 5 cm/s. at the instant when the radius of the circular wave is 8 cm, how fast is the enclosed area increasing?

Q24. If $\vec{a}, \vec{b}, \vec{c}$ are three vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ and $|\vec{a}| = 3, |\vec{b}| = 4\ |\vec{c}| = 5$

Find the value of $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}$.

OR

If $\vec{a} = -3\vec{i} + 2\vec{j}$ and $\vec{b} = -2\vec{i} - 2\vec{j}$ find unit vector in the direction of $3\vec{a} + 2\vec{b}$

Q25. Find equation of line through (2, -1, 1) and parallel to the line whose equation is $\frac{2x-1}{2} = \frac{y+5}{3} = \frac{1-z}{6}$

SECTION C

(THIS SECTION COMPRISES OF SHORT ANSWER TYPE QUESTION (SA) OF 3 MARKS EACH)

Q26. Prove that a Relation R in a Set Z (set of Integers) defined by

R= {(x, y): $|x-y|$ is a multiple of 5 } is an equivalence relation.

OR

Show that the function f : R-{2/3} $\rightarrow$ R -{2/3} defined by $f(x) = \frac{4x+3}{6x-4}$ is bijective function. Prove with justification.

Q27. If cosy = x cos(a + y), with cosa $\neq \pm 1$, Prove that $\frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$.

Q28. Find: $\int \frac{x+3}{\sqrt{5-4x-x^2}} dx$

Q29. Evaluate: $\int_0^{\pi} \frac{x \sin x}{1+\cos^2 x} dx$

OR

Evaluate: $\int_1^4 \left[|x-1| + |x-3|\right] dx$

Q30. .Find the particular solution of the differential equation $x^2 dy = (2xy + y^2) dx$, , given that $y(1) = 1$.

OR

Solve the differential Equation: $\cos^2 x \frac{dy}{dx} + y = \tan x$

Q31. The probabilities of A, B, C solving a problem are $\frac{1}{3}, \frac{2}{7}$ and $\frac{3}{8}$ respectively. If all the 3 try to solve the problem simultaneously Find the probability that Exact one solve the problem.

OR

Let X denote the number of colleges where you will apply after your results and P(X=x) denotes your probability of getting admission in x number of colleges. It is given that

$$P(X=x) = \begin{cases} kx \text{ if } x = 0 \text{ or } 1 \\ 2kx \text{ if } x = 2 \\ k(5-x) \text{ if } x = 3 \text{ or } 4 \end{cases}$$ Where k is constant.

Find the value of k

What is the probability that you will get admission in two colleges exactly?

SECTION D

(This section comprises of long answer-type question (LA) of 5 marks each)

Q32. Use product $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ to solve the system of equations:

$x - y + z = 4,\ x - 2y - 2z = 9$ and $2x + y + 3z = 1$

OR

Solve the following system of equations, using matrices:

$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4,\quad \frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1,\ \frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$

Q33. Using integration compute the area of the region bounded by the lines: $x + 2y = 2$, $y - x = 1$ and $2x + y = 7$.

OR

Prove that the curves $y^2 = 4x$ and $x^2 = 4y$ divide the area of the square bounded by x = 0, x = 4, y = 4 and y = 0 into three equal parts.

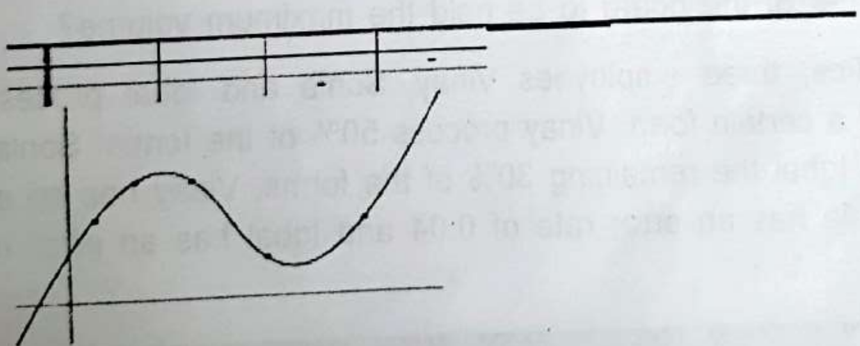
Q34. A man rides his motorcycle at the speed of 25 km/hour. He has to spend Rs 2 per km onpetrol. If he rides it at a faster speed of 40 km/hour, the petrol cost increases to Rs 5 per km. He has atmost Rs 100 to spend on petrol and one hour's time. He wishes to find the maximum distance that he can travel. Formulate this problem as a linear programming problem and solve graphically.

Q35. Find shortest distance between the lines whose vector equations are $\vec{r} = (1-t)\hat{i} + (t-2)\hat{j} + (3-2t)\hat{k}$ and $\vec{r} = (s+1)\hat{i} + (2s-1)\hat{j} - (2s+1)\hat{k}$

SECTION E

(This section comprises of 3 case study/ passage based questions of 4 marks each with two sub-parts. First two case study questions have three sub-parts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study question has two sub-parts of 2 marks each.)

Q36. Case-Study I: Read the following and answer the question given below.



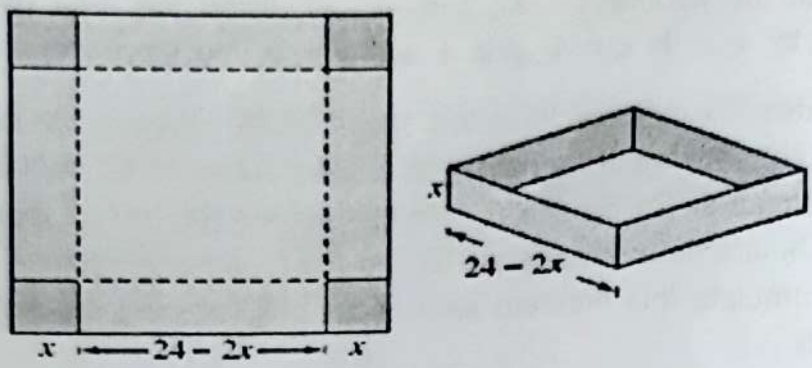
A particle is moving on a curve given by the function

$f(x) = x^4 - 62x^2 + mx + 9, 0 \le x \le 2$, where m is constant. It is given that particle attains its maximum value at x=1

(i) Is function differentiable at x = 1? Justify your answer.

(ii) Find value of m.

(iii) Is f '(x) differentiable in (0 , 5) if yes Find the value of f "(4) .



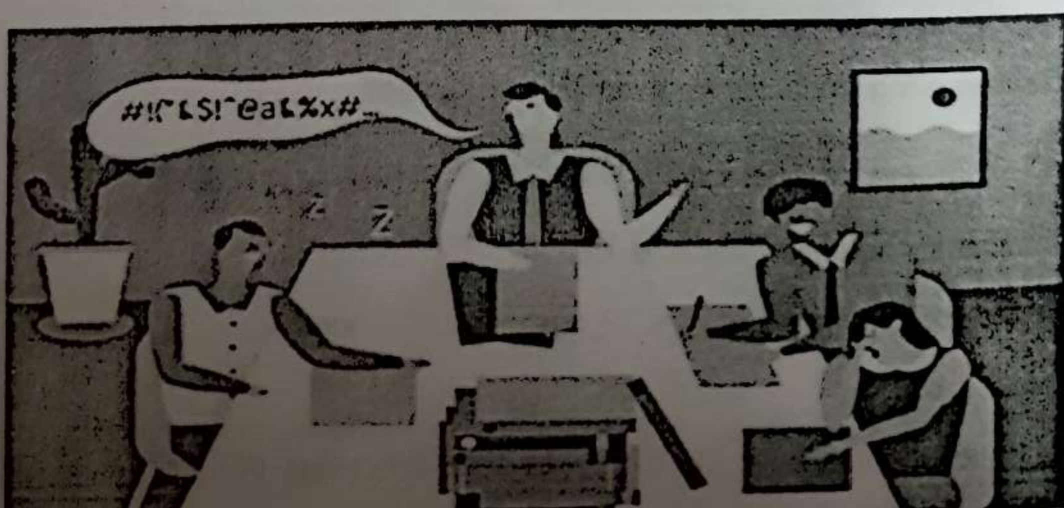
Q37. A man has an expensive square shape piece of golden board of size 24 cm is to be made into a box without top by cutting from each corner and folding the flaps to form a box.

(i) What is the Volume of open the box formed by folding up the flap:

(ii) In the first derivative test, if $\frac{dy}{dx}$ changes its sign from positive to negative as x increases through c1, then which value attains by the function at $x = c_1$

(iii) What should be the side of the square piece to be cut from each corner of the board to be hold the maximum volume?

Q38. In an office, three employees Vinay, Sonia and Iqbal process incoming copies of a certain form. Vinay process 50% of the forms. Sonia processes 20% and Iqbal the remaining 30% of the forms. Vinay has an error rate of 0.06, Sonia has an error rate of 0.04 and Iqbal has an error rate of 0.03



Based on the above information answer the following:

(i) What is the conditional probability that an error is committed in processing given that Sonia processed the form?

(ii) The manager of the company wants to do a quality check. During the inspection, he selects a form at random from the days' output of processed forms. If the form selected at random has an error, Then What is the probability that the form is NOT processed by Vinay?

ANSWER

SECTION-A

1. (C) Transitive
2. (a) One-One
3. (d) $-\frac{x}{2}$
4. (d) $\frac{16}{5}$
5. (c) 8
6. (c) 8
7. (d) 1
8. (b) 2
9. (b) 0
10. (c) $-\frac{2x}{1+x^4}$
11. (c) 1
12. (b) 0
13. (c) 4
14. (c) $\pm \frac{1}{\sqrt{3}}$
15. (b) 5 or -23
16. (b) 3
17. (c) 2, -3, 6
18. (a) $\frac{1}{3}$
19. (d)

20. (a)

SECTION-B

21. $\frac{3\pi}{4}$

OR f(1.1)=f(1.2)=f(1.5)...so many one

22. $(AB - BA)^T = -(AB-BA)$

23. 40 $cm^{2/}s$

24. -25

OR $\frac{1}{\sqrt{29}}(-5\bar{i} + 2\bar{j}$

25. $\frac{x-2}{1} = \frac{y+1}{3} = \frac{1-x}{6}$

26. Prove Equivalence relation

27. Prove $\frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$

28. $-\sqrt{5-4x-x^2} + \sin^{-1}\frac{x+2}{3}C$

29. $\frac{\pi^2}{4}$ 29.OR 7

30. 2y=x(x+y)

Or $y = \tan x - 1 + ce^{-\tan x}$

31. $\frac{75}{68}$ OR $k = \frac{1}{8}, P(x) = 2) = \frac{1}{2}$

32. $B^{-1} = \frac{1}{8}A, x = 3, Y = -2, Z = -1$

OR $|A| = 1200 x = 2, Y = 3, Z = 5$

33. Required Area 6 Sq. units

OR Ar(I) = Ar(II) = Ar(III)= 16/3

34. Max(Z)= x + y , Constraints: $\frac{x}{25} + \frac{y}{40} \le 1, 2x + 5y \le 100, x, y \ge 0$

Max(Z)= 30km when x=50/3 & y= 40/3

35. S.D = $\frac{8}{\sqrt{29}}$ units

36. (i) Diff at x= 1 (ii) m= 120 (iii) f''(4) = 68

37. (i) V = $x(24-2x)^2$

(ii) Local max. at C_1

(iii) Side of Square. 4 cm.

38. (i) $P\left(\frac{E}{E_1}\right) = 0.04$ (ii) P(Not processed by Vinay= $\frac{17}{47}$